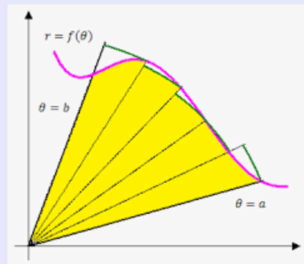
Calculus II

Lecture 6



Feb 19-8:47 AM

$$\lim_{x \to 1} \frac{x^3 - 2x^2 + 1}{x^3 - 1} = \frac{1^3 - 2(1)^2 + 1}{1^3 - 1} = \frac{0}{0} \quad \text{I.F.}$$

Calc. I

$$\lim_{x \to 1} \frac{x^3 - 2x^2 + 1}{x^3 - 1} = \lim_{x \to 1} \frac{\cancel{(x-1)}(x^2 - x - 1)}{\cancel{(x-1)}(x^2 + x + 1)}$$

$A^3 - B^3$

$1 \rfloor$	1	-2	0	1
		1	-1	-1
	1	-1	-1	0

$$= \lim_{x \to 1} \frac{x^2 - x - 1}{x^2 + x + 1} = \frac{1^2 - 1 - 1}{1^2 + 1 + 1} = \boxed{\frac{-1}{3}} \checkmark$$

Calc. II

$$\lim_{x \to 1} \frac{3x^2 - 4x}{3x^2} = \frac{3(1)^2 - 4(1)}{3(1)^2} = \boxed{\frac{-1}{3}} \checkmark$$

$$\lim_{x\to 0}\frac{e^{2x}-1}{\sin x}=\frac{e^0-1}{\sin 0}=\frac{1-1}{0}=\frac{0}{0}\quad \text{I.F.}$$

$$\lim_{x\to 0}\frac{2e^{2x}}{\cos x}=\frac{2e^0}{\cos 0}=\frac{2}{1}=\boxed{2}$$

$$\tanh x=\frac{e^x-e^{-x}}{e^x+e^{-x}}$$

$$\lim_{x\to 0}\frac{\tanh x}{\tan x}=\frac{\tanh 0}{\tan 0}=\frac{\frac{e^0-e^0}{e^0+e^0}}{0}=\frac{\frac{1-1}{1+1}}{0}=\frac{\frac{0}{2}}{0}=\frac{0}{0}\quad \text{I.F.}$$

$$\lim_{x\to 0}\frac{\tanh x}{\tan x}=\lim_{x\to 0}\frac{\operatorname{sech}^2 x}{\sec^2 x}=\frac{\operatorname{sech}^2 0}{\sec^2 0}=\frac{1}{1}=\boxed{1}$$

$$\operatorname{sech} x=\frac{1}{\cosh x}=\frac{1}{\frac{e^x+e^{-x}}{2}}\qquad \operatorname{sech} 0=\frac{1}{\frac{e^0+e^0}{2}}=\frac{1}{\frac{2}{2}}=1$$

$$\lim_{x\to\infty}\frac{(\ln x)^2}{x}=\frac{\infty^2}{\infty}=\frac{\infty}{\infty}\quad \text{I.F.}$$

$$\lim_{x\to\infty}\frac{2\cdot\ln x\cdot\frac{1}{x}}{1}=\lim_{x\to\infty}\frac{2\ln x}{x}=\frac{\infty}{\infty}\quad \text{I.F.}$$

$$=2\lim_{x\to\infty}\frac{\frac{1}{x}}{1}=2\lim_{x\to\infty}\frac{1}{x}=2\cdot 0=\boxed{0}$$

$$\lim_{x\to\infty} x\sin\frac{\pi}{x} = \infty\cdot 0 \qquad \text{I.F.}$$

$x\to\infty$ $\quad \frac{\pi}{x}\to 0$

$\sin\frac{\pi}{x}\to 0$

$$\lim_{x\to\infty} x\sin\frac{\pi}{x} = \lim_{x\to\infty}\frac{\sin\frac{\pi}{x}}{\frac{1}{x}} = \frac{0}{0} \quad \text{I.F.}$$

$$= \lim_{x\to\infty}\frac{\cos\frac{\pi}{x}\cdot\frac{-\pi}{\cancel{x^2}}}{\frac{-1}{\cancel{x^2}}}$$

$x\to\infty$

$\frac{\pi}{x}\to 0$

$\cos\frac{\pi}{x}\to 1$

$$= \pi\lim_{x\to\infty}\cos\frac{\pi}{x}$$

$$= \pi\cdot 1 = \boxed{\pi}$$

$$\lim_{x\to\infty} x\tan\frac{1}{x} = \infty\cdot 0$$

$$= \lim_{x\to\infty}\frac{\tan\frac{1}{x}}{\frac{1}{x}} = \frac{0}{0} \quad \text{I.F.}$$

$$= \lim_{x\to\infty}\frac{\sec^2\frac{1}{x}\cdot\frac{\cancel{-1}}{\cancel{x^2}}}{\frac{\cancel{-1}}{\cancel{x^2}}} = \lim_{x\to\infty}\sec^2\frac{1}{x}^{\to 0} = \boxed{1}$$

$$\lim_{x\to1}\left(\frac{x}{x-1}-\frac{1}{\ln x}\right) = \infty-\infty \quad \text{I.F.}$$

$$=\lim_{x\to1}\left[\frac{x\ln x - x+1}{(x-1)\ln x}\right] = \frac{0}{0} \quad \text{I.F.}$$

$$=\lim_{x\to1}\frac{1\cdot\ln x + x\cdot\frac{1}{x} - 1}{1\cdot\ln x+(x-1)\cdot\frac{1}{x}} = \lim_{x\to1}\frac{\ln x}{\ln x+\frac{x-1}{x}} = \frac{0}{0}$$

$$=\lim_{x\to1}\frac{\frac{1}{x}}{\frac{1}{x}+\frac{1}{x^2}}$$

$$=\frac{\frac{1}{1}}{\frac{1}{1}+\frac{1}{1^2}} = \boxed{\frac{1}{2}}$$

$$\frac{d}{dx}\left[\frac{x-1}{x}\right] = \frac{d}{dx}\left[\frac{x}{x}-\frac{1}{x}\right] = \frac{d}{dx}\left[1-\frac{1}{x}\right] = \frac{1}{x^2}$$

$$\lim_{x\to0^+}(4x+1)^{\cot x} = 1^{\infty} \quad \text{I.F.}$$

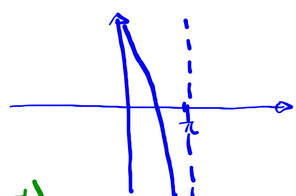
$x\to0^+$, $4x+1\to1$

$\cot 0^+\to\infty$

$$= e^{\boxed{4}}$$

$$y=(4x+1)^{\cot x}$$

$$\ln y = \ln(4x+1)^{\cot x}$$



$$\ln y = \cot x\,\ln(4x+1)$$

$$\lim_{x\to0^+}\cot x\,\ln(4x+1) = \infty\cdot0$$

$$\lim_{x\to0^+}\frac{\ln(4x+1)}{\tan x} = \frac{0}{0}$$

$$\lim_{x\to0^+}\frac{\frac{4}{4x+1}}{\sec^2 x} = \frac{\frac{4}{1}}{1} = \boxed{4}$$

$$\lim_{x\to a^+} \frac{\cos x \ln(x-a)}{\ln(e^x-e^a)} = \frac{\infty}{\infty}$$

$x \to a^+$

$x-a \to 0^+$

$\ln(x-a) \to -\infty$

$e^x - e^a \to 0^+$

$\ln(e^x-e^a) \to -\infty$

$= \cos a \cdot 1 = \boxed{\cos a}$

Calc I

$$\lim_{x\to a}\left[f(x)\, g(x)\right] = \lim_{x\to a} f(x) \cdot \lim_{x\to a} g(x)$$

If $\lim_{x\to a} f(x)$ & $\lim_{x\to a} g(x)$ exist.

$$\boxed{\lim_{x\to a^+} \cos x = \cos a}$$

$$\lim_{x\to a^+} \frac{\ln(x-a)}{\ln(e^x-e^a)} = \frac{-\infty}{-\infty}$$

$$\lim_{x\to a^+} \frac{\frac{1}{x-a}}{\frac{e^x}{e^x-e^a}} =$$

$$\lim_{x\to a^+} \frac{e^x-e^a}{e^x(x-a)} = \frac{0}{0}$$

$$= \lim_{x\to a^+} \frac{e^x}{e^x(x-a)+e^x\cdot 1} = \lim_{x\to a^+} \frac{\cancel{e^x}}{\cancel{e^x}(x-a+1)}$$

$$= \lim_{x\to a^+} \frac{1}{x-a+1} = \frac{1}{a-a+1} = 1$$